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OPTIMAL FUNDING OF STATE EMPLOYEE PENSION SYSTEMS

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ABSTRACT

Public pension funds are a significant, and rapidly growing, financial force in the United States. However, the lack of a consensus on an appropriate funding level is apparent from the wide diversity of funding levels currently maintained. This research proposes a financial standard for public pension plan funding that depends on the current pension obligation and the respective growth rates of pension expenses and the tax base, and then compares the optimal funding levels based on this standard with actual funding levels by state. Based on this approach, funding levels should vary by state based on economic conditions. However, many states are funding public pension plans at levels well below the optimal values, which creates the potential for serious problems in the future.

INTRODUCTION

The funding of public pension plans is a major political and economic issue facing state and local government units. Whereas federal regulations establish funding requirements and provide insurance coverage in the case of insolvency for private pension plans, no such guidelines or insurance applies to public pension plans. Pension benefits for public employees can be fully funded when the service is provided, paid for only after the retirement of the workers or funded at some interim level. The funding strategy adopted will affect current and future tax rates, the willingness of public employees to exchange current salary for promised future retirement benefits, the investment rating of debt issued by the government unit, and even property values within the area.

The funding ratio of pension plans is the ratio of accumulated assets to the present value of the cost of benefits that have already been earned. Actual funding ratios of different public pension plans currently run the gamut from approximately zero (i.e.,

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Pay-As-You-Go funding where all pension costs are deferred to the time when benefits must be paid) to over 100 percent (which not only covers the entire cost of all benefits already earned, but also prefunds benefits that will be earned in the future or future benefit increases).

This study focuses on developing an appropriate funding level target for public pension plans. Basic assumptions are made that lead to the conclusion that the optimal tax policy is for the pension tax to be a constant percentage of taxable income for every period. Initially, a two-period model is developed that incorporates the relevant factors in a simple model and examines the comparative statics. A multi-period model is then developed to examine the optimal funding paths under various economic assumptions. The results indicate that the relationship between the growth rates of pension costs and tax base is the critical element in determining the appropriate funding level. Finally, the optimal funding paths to go from the current actual funding levels to full funding are determined for each state.

LITERATURE REVIEW

State and local government pension plan assets represent a significant component of the nation's total pension plan assets. According to the American Council of Life Insurance (1996), at the end of 1994 the nation's pension assets totaled approximately $5 trillion. Of this, private pension plans held $3.1 trillion while state and local government pension plan assets totaled $1 trillion and provided coverage for over 18 million workers.[1] The majority of these public retirement plans are defined benefit plans, and they represent the primary source of retirement income for state and local government employees. In spite of the importance of these plans, there has been a well-acknowledged lack of research on the funding of state and local government pension plans (Congressional Research Service, 1990; Dulebohn, 1995; Mitchell and Smith, 1994).

These public pension plans have experienced a tremendous growth in assets since 1979; at that time the aggregate assets were approximately $169 billion (American Council, 1996). During this period of growth, a number of changes have occurred that have drawn attention to state and local plan funding. First, since the late 1970s a number of state and municipal governments have faced periods of budgetary stress, characterized by increasing expenditure requirements and decreasing revenues. Second, and related, there has been an increased use of public plan assets, sometimes facilitated by the altering of the actuarial assumptions used to compute public pension obligations, to help balance state and local budgets (Mitchell and Smith, 1994; General Accounting Office, 1992; House Select, 1992; Flanagan, 1993). Third, the composition of the workforce in state and local governments is aging; currently over 75 percent of state and local retirement plan members are 40 years or older and many are expected to retire before the age of 60 (Greenwich, 1993; Posner, 1993). Together these factors have contributed to a concern about the adequacy of state and local pension plan funding.

[1] The remaining assets are held in federal government-administered plans.

A large amount of research has addressed the appropriate funding level for private pension plans. Early research on private pension plan funding was conducted in the absence of empirical work. More recently, research has been facilitated by the availability of data on private pension plans as a result of Title 1 of the Employees' Retirement Income Security Act of 1974 (ERISA) which requires private defined benefit pension plans to provide periodic disclosure of their funding status to the IRS. In 1988 the Pension and Welfare Benefits Administration of the Department of Labor began making edited files of the data on the universe of private pension plans (that submit the Form 5500 to the IRS) available to researchers. Before this, the Pension and Welfare Benefits Administration provided edited sample files first in 1977 and then from 1981 through 1987. In general, the results of research on private pension plans have favored either full funding to take advantage of the tax benefits associated with pensions or minimum funding to maximize the subsidy of pension insurance (Black, 1980; Feldstein and Seligman, 1981; Harrison and Sharpe, 1987; Lewis and Pennacchi, 1994; Sharpe, 1976; Tepper, 1981; Treynor, 1977; Westerfield and Marshall, 1983).

In contrast to private defined benefit pension plans, much less research has been conducted on the funding of public pension plans. Research efforts have been hindered by the lack of data since there is no federally mandated disclosure requirement, or regular and systematic compilation of data, on state and local pension funding and liability status. The prevailing view of public pension plan funding is that public plan sponsors need to follow actuarially determined advanced funding policies, similar to that required of private defined benefit plans by ERISA (cf., House Committee, 1978; Inman, 1982; Inman, 1985; Congressional Research Service, 1990). Most of the empirical research on public pension plan funding began in the wake of a comprehensive Congressional study of state and local public pension plans in 1978. This study concluded that many of these pension systems were inadequately funded and they lacked uniform fiduciary and actuarial funding standards (General Accounting Office, 1979; House Select, 1978). The majority of studies conducted since that report have been descriptive analyses of funding levels among state and local public pensions, although a few have sought to develop and test models that can contribute to understanding the dynamics of public pension plan funding or that can inform policy makers.

A number of descriptive analyses have reported the funding levels of public pension plans (Dulebohn, 1995; GAO, 1992; Testin, 1992; Church, 1992; Zorn, 1993). These studies have reported similar funding ratios of approximately 86 percent for FY 1992 for public plans and have indicated that many state and local public pension plans have experienced improvement in funding levels over time as a result of following actuarially sound funding standards and experiencing robust investment returns. In spite of an overall improvement in funding condition, it is estimated that only 30 percent of the major state and local public retirement plans are funded at 100 percent or higher (Dulebohn, 1995).

Several studies have investigated why public pension plans are not fully funded. This research has argued that the unfunded liability of public pension plans represents a deferral or shift of labor costs to the future. Past taxpayers are the beneficiaries of underfunding because current or future taxpayers are responsible for paying the unfunded retirement benefits workers earned in the past. Mumy (1978) developed a two-period model that considers the investment rate of return experienced from advanced funding with the borrowing costs of Pay-As-You-Go funding. Mumy's model did not specify an optimal funding level but stated that it was dependent on preferences for labor services between period one and period two, the time profile of revenue flows, and pension size. Further, Mumy's model assumed that taxpayers remain in a local community indefinitely.

Epple and Schipper (1981) empirically addressed the question of whether pension underfunding is a result of a rational strategy employed to pass on costs to future residents or of tax-smoothing. In their model, they estimated the degree of capitalization of pension underfunding and considered the potential benefit of leveling the tax burden of citizens by adjusting the pension funding level to reflect changes in taxable income. Their results supported the tax-smoothing model as an explanation for underfunding, although they did not consider the implications of taxpayers moving.

Inman (1981a, b, 1982) noted that because of the mobility of society, taxpayers often move and therefore do not remain in a certain locale long enough to bear the cost of unfunded benefits. Because of this, pension underfunding shifts a portion of the current labor cost to future residents while current residents reap the labor benefit. In his examination of public pension underfunding, Inman developed two models, a stayer and a mover model, to account for whether taxpayers remain in the locale or relocate before the pension obligation becomes due.

An underfunded pension represents a loan from current workers to taxpayers at the current rate of return on the pension fund (Inman, 1982). In the stayer model, current taxpayers who receive the benefits of the loan, in the form of lower taxes, stay and pay employees' future retirement benefits in the form of higher taxes. In the mover model, the current taxpayers who receive the benefits of the loan, in the form of lower taxes, move in order to avoid repaying the loan in the form of higher taxes. The stayer model assumes that public employee wage demands will be affected by the funding level adopted. In the mover model, the degree to which pension underfunding is capitalized in property values determines funding strategy. If property values do not reflect the full level of underfunding, then it is considered optimal to fund the pension plan minimally and move when the obligation becomes due. This situation, however, assumes that the mover can relocate to another jurisdiction that has not been underfunding its pension plan.

Support for the mover over the stayer model was provided by Inman's (1982) examination of police and fire services in 60 large U.S. cities and Inman and Albright's (1987) study of teacher pension plans in 48 states. Both of these studies found a strong bias in state and local governments towards Pay-As-You-Go funding. For instance, Inman found that lump-sum pension aid from states, designed to fund current pension liabilities and preclude cheating by shifting the liabilities to the future, actually resulted in a reduction of local contributions. According to Inman, the underfunding strategy typically followed by state and local governments and the high mobility of taxpayers together pressure public pension sponsors to shift the cost of pension benefits to future residents.

More recently, Mitchell and Smith (1994) examined determinants of plan funding, noting wide variations in funding practices among state and local public pension plans. Specifically, they found that variations in plan funding were partially explained by past funding practices, unionization, and fiscal pressure experienced by the sponsoring government.

A primary question that has not been addressed by research on state and local public pension plans is what the optimal funding level of state and local public pensions should be. Although most public pension plans are funded below 100 percent, there is little research to support, or refute, this funding strategy. In addition, while it is generally held that a defined benefit plan is fully funded when the assets are 100 percent of termination liability, because of the uncertainty associated with pension valuations and the increasing trend of pension liabilities, higher target ratios have been suggested (cf., U. S. Congress, 1987). Currently no model exists to guide state and local public pension funding policy. The goal of this study is to meet this need by generating a usable funding model for public pension plans.

TWO-PERIOD MODEL

In this section, a two-period model of the optimal funding decision is developed. By considering a simple model dealing with pension funding, the effect of economic and demographic variables on the optimal funding decision can be examined. In this model, individuals work for one period and are retired for the next period. Based on the average number of years worked and life expectancy after retirement for a major representative public pension fund, each period in this model is represented by 20 years.[2] A fund's sponsoring government needs to collect taxes in order to pay the pension obligation, but it can elect to raise this revenue at the beginning of the first period or the beginning of the second period. Following the approach of Epple and Schipper (1981), this model assumes that taxpayers' utility functions are concave and the interperiod adjustment factor (or the discount rate used to account for timing differences) is the same as per capita income growth. Thus, utility is maximized when taxes are a constant proportion of income. (Appendix A demonstrates this relationship for lognormal utility in a multiperiod case.)

The following simplifying assumptions are made:

1. The funding level of the public employee retirement system is the key variable for policymakers, and the other costs, such as current wages or capital expenditures, are fixed.
2. All pension liabilities will be paid off in period two; therefore, at the beginning of period two, the taxpayer is responsible for the underfunded portion of the pension obligation of period one, if any, in addition to the full pension obligation of period two.
3. In the case of advance funding, the government can save period one's revenues by placing them in a pension-fund investment to accumulate at an interest rate r.

[2] The State Universities Retirement System (SURS), one of Illinois' five state sponsored public pension funds, indicates that the average number of years of service is approximately 20 years, and the life expectancy of retirees at the average retirement age is also approximately 20 years (SURS, 1991).

4. The pension obligation grows at a rate of e.
5. The population grows at a certain rate d during the period, and the per capita income of residents grows at a certain rate g.
6. The utility function for state taxation is such that period two's utility is maximized when taxes are a constant proportion of taxable income each period.

The following notation will be used throughout (subscripts indicate the period):

Y = gross income of the representative taxpayer

τ = marginal state tax rate of the representative taxpayer

r = interest rate on pension fund investments

N = the number of taxpayers in the state

p = total pension payable at retirement to public employees

e = pension growth rate, $\left(\frac{P_2}{P_1} - 1\right)$

g = income growth rate, $\left(\frac{Y_2}{Y_1} - 1\right)$

d = population growth rate, $\left(\frac{N_2}{N_1} - 1\right)$

x = optimal funding ratio for period one

In this model, since the sponsoring government wants to equalize the pension tax rate for each period, the optimal funding level would be determined by the following two equations:

$$\frac{p_1}{1+r} x = \tau N_1 Y_1 \tag{1}$$

and

$$(1-x)p_1 + \frac{p_2}{1+r} = \tau N_2 Y_2. \tag{2}$$

Equation (1) indicates the tax revenue to fund the pension liabilities earned in the first period. Equation (2) indicates that, at the beginning of the second period, the state must fund the remainder of pension liabilities from the first period as well as the full pension obligation for the second period. From the above equations, the following funding equation can be obtained.

$$x = \frac{(1+r)+(1+e)}{(1+d)(1+g)+(1+r)} \tag{3}$$

Equation (3) shows how the optimal funding ratio for a state's pension plan is a function of interest rates, population and per capita income growth rates, and pension growth rate.

From equation (3), the following results hold straightforwardly:

1. In a static world with $g = d = r = e = 0$, full funding is always optimal, i.e., $x = 1$.
2. When $(1+e) = (1+g)(1+d)$, full funding is always optimal, i.e., $x = 1$.
3. When $(1+e) > (1+g)(1+d)$, overfunding is optimal, i.e., $x > 1$.
4. When $(1+e) < (1+g)(1+d)$, partial funding is optimal, i.e., $0 < x < 1$.
5. The greater the magnitude of $(1+g)(1+d) - (1+e)$, the lower the optimal funding level.

Application of the Model

For an example, the values for Illinois are used to illustrate this approach. The pension growth rate e can take on any number of values. For this example, it is assumed that pension growth reflects the growth in the number and the average salary of public employees. This ignores the effect of a change in the age composition of public employees, a change in pension accrual rates, or other benefit changes (lowering the retirement age, reducing the number of years of service required for full benefits, etc.). Other values will be tested later in this paper. Over the period from 1980 through 1992, the public employee growth in Illinois averaged 0.592 percent per year and the average salary growth rate 5.121 percent per year (Bureau of the Census (1980 and 1992)). Combining these factors and calculating the overall effect for 20 years is $[((1.00592)(1.05121))^{20} - 1 = 2.0553]$; thus $e = 2.0553$. Over the same period, the population in Illinois grew at an annual rate of 0.135 percent, which would be 2.73 percent over 20 years. Thus, $d = .0273$. Over the same time period, the per capita income grew at an annual rate of 5.956 percent, which would be 218.06 percent over 20 years; thus, $g = 2.1806$. Both of these values are used as the expected increases over the next twenty-year period to determine the optimal funding level.

The investment allocation of the typical public pension plan is approximately equally weighted in equities and bonds.[3] The expected return on a portfolio allocated along these lines would be approximately 8 percent.[4] This would produce a twenty-year return of 366.10 percent. Thus, $r = 3.6610$. Calculating the optimal funding ratio, x, then involves:

$$x = \frac{(1+r)+(1+e)}{(1+d)(1+g)+(1+r)} =$$

$$\frac{(1+3.6610)+(1+2.0553)}{(1+0.0273)(1+2.1806)+(1+3.6610)} = 0.973$$

Based on these assumptions, the optimal funding level for Illinois would be 97.3 percent. It is less than 100 percent, since the growth in the tax base exceeds the growth in pension costs, $(1+d)(1+g)>(1+e)$.

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[3] See *Pensions and Investments*, Jan. 24, 1994, p. 27, for details.

[4] For this study, the pension return is assumed to be 8 percent per year. This is in line with the average pension plan interest rate assumption in 1992, as well as the long-term (1926-1995) weighted average return on a portfolio invested equally in common stocks of large companies and long-term corporate bonds (Ibbotson, 1996).

Comparative Statics

Although the model developed here is a simple representation of state government financial decision-making on pension funding, it does allow the calculation of comparative-statics which provide a basis for empirical testing. The obvious results are:

1. An increase in the income growth rate, g, leads to a decrease in the optimal pension funding level, x.
2. An increase in the population growth rate, d, also leads to a decrease in the optimal pension funding level, x.
3. An increase in interest rates, r, leads to an increase or a decrease in the optimal funding level, x, depending upon the relative magnitude of tax base growth rate and pension growth rate.
4. An increase in the pension growth rate, e, also leads to an increase in the optimal funding level, x.

From the optimal funding level in equation (3), the following derivatives can be obtained:

$$\frac{\partial x}{\partial g} = -\frac{[(1+r)+(1+e)](1+d)}{[(1+d)(1+g)+(1+r)]^2} < 0,$$

$$\frac{\partial x}{\partial d} = -\frac{[(1+r)+(1+e)](1+g)}{[(1+d)(1+g)+(1+r)]^2} < 0,$$

$$\frac{\partial x}{\partial r} = \frac{(1+d)(1+g)-(1+e)}{[(1+d)(1+g)+(1+r)]^2} \leq or > 0$$

$$\frac{\partial x}{\partial e} = \frac{1}{(1+g)(1+d)+(1+r)} > 0.$$

Thus, as g or d increase, it is better for the taxpayers to fund the pension system less adequately in the first period and postpone the pension funding until the second period. This is intuitively plausible since an increase in g or d means that period two state income (or tax base) increases because of either increased period-two per capita income or an increase of the number of the residents. Thus, the tax burden is lessened. However, the relationship between the interest rate, r, and optimal funding level depends on the relative magnitude of tax base growth rate $[(1+g)(1+d)]$ and pension growth rate $(1+e)$. The optimal funding level increases as the interest rate increases only when the tax base is growing faster than the pension obligation. In this case, by funding the pension plan more adequately, the greater investment income reduces the following period's tax burden. But when the pension cost is growing more rapidly than the tax base, the optimal funding level decreases as the interest rate increases. This occurs because by discounting the pension obligation in period two at a greater interest rate, the tax saving in period two becomes greater.

MULTI-PERIOD MODEL

In this section, a multi-period version of the optimal pension funding decision is developed. The multi-period model will be premised on the assumption that the pension plan is a long-term commitment between the employer and employees. It will be assumed that all employees work for 20 years and then collect a pension for 20 years.[5] Thus, employees hired in the first year will start collecting pension benefits in year 21, employees hired in the second year will start receiving pension benefits in year 22, and so forth. The pension benefit is assumed to be a portion of the final year's salary times the number of years worked.

As in the previous two-period model, it will be assumed that the utility of taxation is maximized when the pension tax rate for every period is equal. Economic and demographic growth rates, pension growth rate, mortality rates, and different working patterns, as well as interest rates, affect the necessary tax rate. Since the time horizon of a public pension system may not be limited, and instead could continue for as long as the sponsoring government exists, the steady state optimal pension funding ratio depends on the time horizon selected.

In order to make the model straightforward, the following simplifying assumptions are made, in addition to the ones listed for the two-period model.

1. The pension system is closed at the final period (not the second period).
2. Pension assets are constructed from each year's pension tax raised and accumulated at an annual interest rate r.
3. Pension liabilities are constructed from each year's accrued benefit and are calculated by discounting future benefit payments at a discount rate q.
4. The number of public employees increases at a certain rate d' each year.
5. The salary of public employees grows at a certain rate g' each year.

The following additional notation is introduced:

$C(t)$ = pension tax paid at the beginning of the year t

$D(t)$ = accrued benefit at the end of the year t

$P(t)$ = benefit payments paid by the pension fund at the end of the year t

$A(t)$ = fund level, i.e., accumulated assets at the end of the year t

$L(\mathrm{t})$ = accumulated liabilities at the end of the year t

$S(\mathrm{t})$ = salary of a public employee at year t

q = discount rate

d' = annual growth rate of the number of public employees

g' = annual growth rate of the salary of public employees

If $t = 0$ is the commencement date of the pension plan, then the following basic relationships apply:

[5] The model was also tested using varying employment patterns and mortality, but the results were not significantly affected (See Oh, 1995).

$$N(t+1) = (1+d)N(t) = (1+d)^{t+1}N(0) \tag{4}$$

$$Y(t+1) = (1+g)Y(t) = (1+g)^{t+1}Y(0) \tag{5}$$

$$S(t+1) = (1+g')S(t) = (1+g')^{t+1}S(0) \tag{6}$$

$$C(t) = \tau Y(t)N(t) \tag{7}$$

$$A(0) = 0$$

and

$$A(t+1) = [A(t)+C(t)][1+r(t+1)] - P(t) \tag{8}$$

$$L(t+1) = L(t)[1+q] + D(t) - P(t) \tag{9}$$

$$x(t) = \frac{A(t)}{L(t)} \text{ for } t = 1, 2, 3, \ldots \tag{10}$$

Fully Employed System

There are employees of different ages and working careers in the system when the pension plan is introduced; the oldest employees, who are assumed to have been working 19 years when the pension plan starts, work one more year and then retire. They are entitled to pension benefits after retirement, even though their earned benefits are small, in this case, 1/20th of their final salary. This benefit will be paid for 20 years. The employees who have been working 18 years work two more years and then receive a pension benefit of 2/20ths of their final salary for 20 years after retirement, and so forth. The total annual pension payment increases as the number of retirees increase. The system will be stabilized 40 years after the system has started, when each retiree will be receiving a benefit based on 20 years of service, and the new retirees each year simply replace the number that die.

The optimal funding strategy is illustrated for a time horizon of 80 years. This time period was selected in part because it allows the steady state to exist for as long as it takes for it to develop. Also, this time horizon is approximately the same as the time frame used to evaluate Social Security funding (75 years).[6]

Accumulated liabilities are divided into two parts, for employees and for retirees. Considering the various growth rates described in the previous section, accumulated liabilities and pension payments across time are expressed as follows (see Appendix B for the derivation):

Accumulated Liabilities for Employees

$$L(t) = \frac{S}{20}\sum_{i=1}^{20}\left(\frac{1}{1+q}\right)^{i}\left[(1+d')(1+g')\right]^{t\pm1}\left[t\sum_{i=1}^{20\pm t}\left(\frac{1+d'}{1+g'}\right)^{i} + \sum_{i=1}^{t\pm1} i\left(\frac{1+d'}{1+g'}\right)^{20\pm i}\right], \text{ for } 1 \le t \le 20,$$

and $L(t+1) = (1+g')(1+d')L(t)$, for $t \ge 21$

[6] Oh (1995) also examined longer time horizons and found similar results.

Accumulated Liabilities for Retirees

$$L(1) = 0$$

$$L(t) = \frac{S}{20}\left\{\sum_{i=1}^{t-1} i[(1+\mathrm{d}')(1+\mathrm{g}')]^{i-1} A_{20-t+i}\right\}, \text{ for } 2 \le t \le 21,$$

$$L(t) = \frac{S}{20}\left\{\sum_{i=1}^{40-t} (t-20+i)[(1+d')(1+g')]^{t-21+i}\right\}A_i + S(t-21)\sum_{i=1}^{t-21}[(1+d')(1+g')]^{19+i} A_{40-t+i},$$
$$\text{for } 22 \le t \le 40$$
$$\text{where } A_i = \text{Present Value of Annuity for } i \text{ years}$$

$$L(t+1) = \{(1+d')(1+g')\}L(t), \text{ for } t \ge 41 \tag{11}$$

Payments to Retirees

$$P(1) = 0$$

$$P(2) = \frac{1}{20}S$$

$$\begin{aligned} P(t) &= P(t-1) + \frac{t-1}{20}S\left[(1+g')(1+d')\right]^{t-2}, 3 \le t \le 21 \\ &= P(t-1) + S\left[(1+g')(1+d')\right]^{t-2} - \frac{t-21}{20}S\left[(1+g')(1+d')\right]^{t-22}, 21 < t \le 40 \\ &= [(1+g')(1+d')]P(t-1), 41 \le t < 80 \end{aligned} \tag{12}$$

The tax rate is determined by setting the sum of the present value of accrued benefits (equation (11)) and payments (equation (12)) equal to the sum of present value of contributions over the entire period considering the interest rate and various growth rates. The optimal funding ratios for each period are determined by equation (10), dividing the accumulated assets (equation (8)) by the accumulated liabilities (equation (9)).

Based on this framework, the optimal funding ratio is calculated over an 80-year time horizon with the demographic and economic values determined from the U.S. national average data between 1980 and 1992. Over this period, the population of the U.S. grew 0.993 percent per year, and per capita personal income grew at 6.046 percent annually; the number of state public employees grew 1.539 percent per year, and average salary grew 5.536 percent per year (Bureau of the Census, 1980, 1992, 1994). Table 1 represents the optimal funding ratios and other variables across the entire 80-year period for a hypothetical pension plan covering all state public workers; the optimal funding ratios are illustrated graphically in Figure 1. In this situation, the optimal funding ratio is initially 1.631 and then gradually decreases to one by the final period. Thus, when instituting a public pension system that does not provide retroactive liabilities, it is optimal to overfund the system initially in recognition of the fact that future benefit payments will increase rapidly as retirees with more service credits will begin receiving benefits in future years.

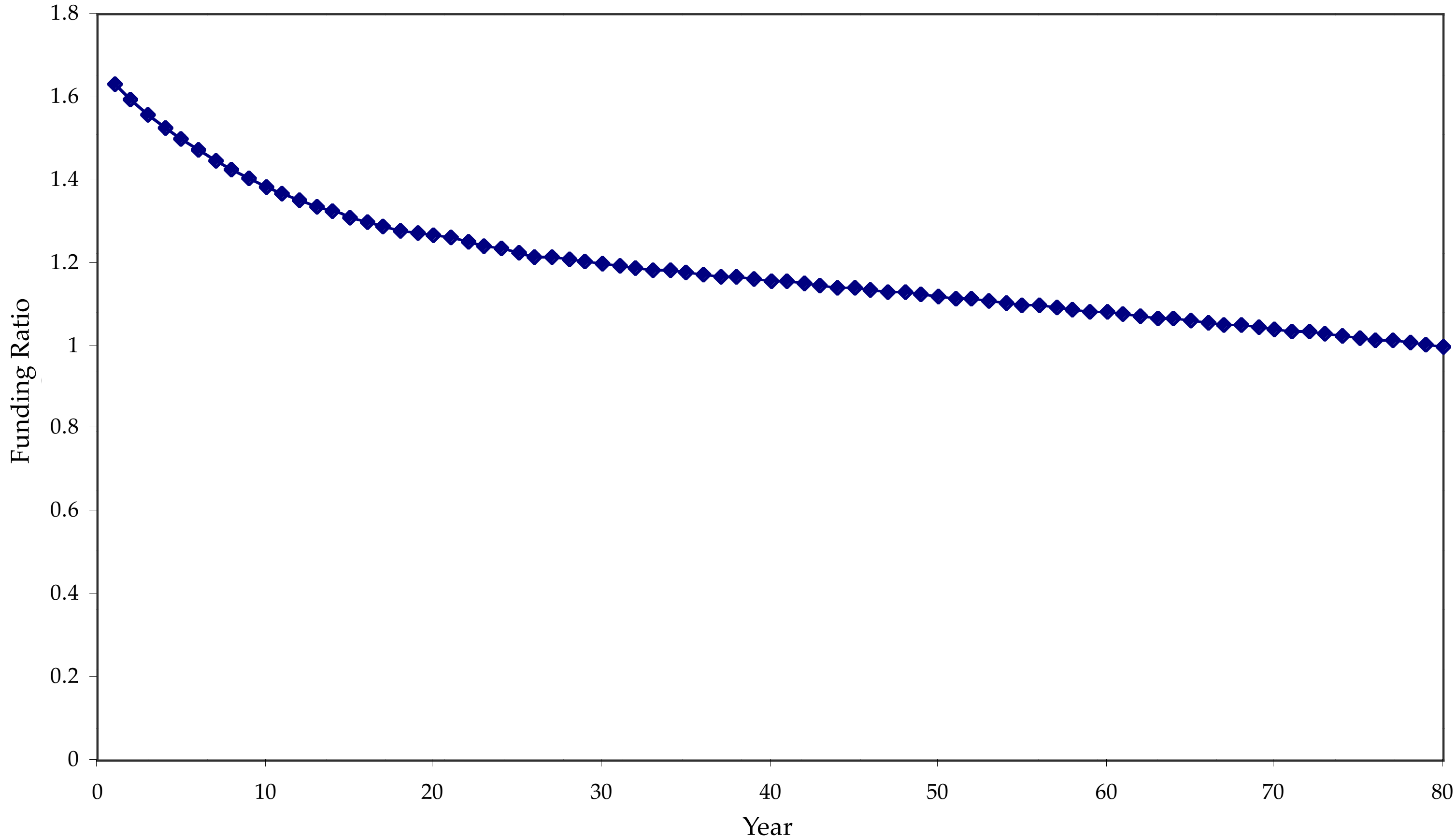
TABLE 1
Optimal Pension Funding Level for 80 Periods
Fully Employed System
Annual Growth Rates (1980–1992)

Population	1.00993	Salary	1.05536
Per Capita Income	1.06046	# of Employees	1.01516
Discount Rate	1.08	Interest Rate	1.08

Year	*Payment*	*Accumulated Liabilities Workers*	*Accumulated Liabilities Retirees*	*Accumulated Liabilities Total*	*Pop.*	*Income*	*Tax*	*Accumulated Assets*	*Funding Ratios*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	0	581	0	581	100	100	877	947	1.631
2	5	1228	48	1276	101	106	939	2032	1.592
3	16	1946	150	2096	102	112	1006	3264	1.557
4	33	2740	311	3051	103	119	1077	4656	1.526
5	58	3612	540	4152	104	126	1153	6216	1.497
6	90	4568	841	5410	105	134	1235	7957	1.471
7	133	5611	1225	6836	106	142	1323	9890	1.447
8	186	6744	1698	8442	107	151	1417	12026	1.424
9	251	7969	2271	10240	108	160	1517	14376	1.404
10	329	9289	2952	12241	109	170	1625	16953	1.385
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
37	12441	86848	101139	187986	143	828	10354	219730	1.169
38	13381	93045	109434	202479	144	878	11089	235904	1.165
39	14371	99685	118371	218056	146	931	11876	253232	1.161
40	15415	106798	128018	234817	147	987	12720	271812	1.158
41	16515	114419	138450	252869	148	1047	13623	291754	1.154
42	17694	122584	149729	272313	150	1110	14590	313157	1.150
43	18957	131332	161926	293257	151	1177	15625	336128	1.146
44	20309	140703	175113	315817	153	1248	16735	360782	1.142
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
77	197495	1368243	2305250	3673493	212	8661	160894	3718029	1.012
78	211588	1465880	2492104	3957984	214	9185	172316	3989985	1.008
79	226687	1570484	2694080	4264564	216	9740	184548	4281809	1.004
80	4837816	1682553	2912400	0	218	10329	197650	0	1.000

Note: Column (2) is equal to salary (initially 100) times units earned by retirees each year (initially 1/20) multiplied by pension growth rate. Column (3) is calculated by discounting the future payments earned by working employees. Column (4) is calculated by discounting the unpaid future payments for retirees. Column (5) is the sum of column (3) and column (4). Column (6) is the initial population (100) times (n–1)th power of (1.0099). Column (7) is the initial income (100) times (n–1)th power of (1.065). Column (8) is column (6) times column (7) times the tax rate. Column (9) is the accumulated value of column (8) with interest minus column (2). Column (10) is the ratio of column (9) to column (5).

Figure 1
Optimal Funding Ratio Under Fully Employed System



Retroactive Liabilities

In practice, many public pension systems, when instituted, provide immediate credit for service before the inception of the pension plans, thus leading to retroactive liabilities. This provides an attractive benefit for long-term employees but generates substantial additional costs for taxpayers. In this section, retroactive liabilities are considered for the fully employed system. With a constant tax rate, accumulated pension liabilities in the earlier periods will exceed the accumulated pension assets, resulting in a very low funding ratio initially.

For determining retroactive liabilities, it is assumed that there are 20 blocks of employees in the system, and each block of employees has earned pension benefits commensurate with their past service as of the commencement of the pension plan. Thus, employees hired 19 years ago who have only one more year, the 20th year, until retirement, are immediately credited with pension benefits equal to 19/20ths of their final salaries. Similarly, employees hired 18 years ago are credited with pension benefits equal to 18/20ths of their final salaries. The employees hired one year ago are credited with pension benefits equal to 1/20th of their final salaries. New employees who have just been hired have not earned any pension benefits yet, since they have not accumulated any years of service. Based on this approach, the retroactive liabilities at time 0 can be calculated as follows (see Appendix C for the derivation):

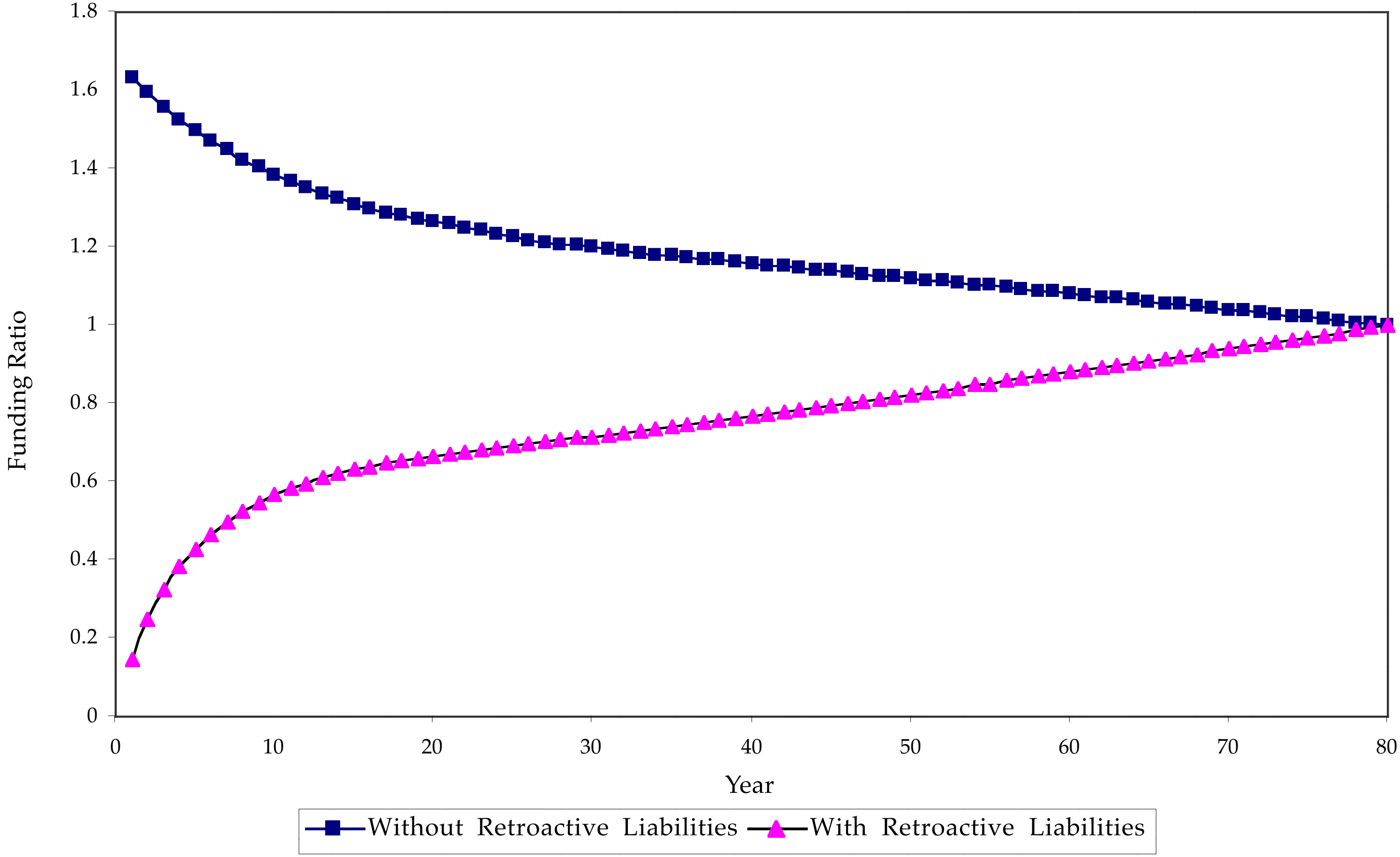
$$L(0) = \frac{S}{20}\left[\frac{1-(1+q)^{-20}}{q(1+q)}\right]\left[\frac{k(k^{19}-1)-19(k-1)}{(k-1)^2}\right], \text{where } k = \frac{1+d'}{1+q}. \tag{13}$$

Annual accrued benefits across time for 80 years are the same as the case of a fully employed system without retroactive liabilities. Since existing employees have already earned pension benefits and are entitled to receive full benefits upon retirement, payments under a retroactive scheme are higher than the case without retroactive liabilities and can be expressed as follows:

$$\begin{aligned} P(t) &= S\left\{\frac{1-[(1+g')(1+d')]^t}{1-(1+g')(1+d')}\right\} \text{for } 1 \le t \le 20 \\ &= (1+g')(1+d')\cdot P(t-1) \text{ for } 21 \le t < 80. \end{aligned} \tag{14}$$

Using this framework, the optimal funding ratios are calculated over an 80-year horizon using the same parameters presented in the previous example (U.S. national average values). Figure 2 illustrates the optimal funding ratios both with and without retroactive liabilities. Unlike the first case, the optimal funding ratio is less than one each year when retroactive liabilities are included. When there are no retroactive liabilities, pension funding through a level tax rate generates more income than is needed to pay benefits in the first few years. The opposite occurs when workers are given credit for past service that was not funded. The cost of paying for the huge unfunded initial liability is then spread over the entire period, which results in underfunding of the pension plans. The constant tax rate with retroactive liabilities is also greater when retroactive liabilities are included than without retroactive liabilities (0.0973 vs. 0.0877). This increase occurs because pension contributions need to cover the initial retroactive liabilities as well as the accrued benefits for each year.

Figure 2
Optimal Funding Ratios With and Without Retroactive Liabilities



Ongoing Employment Pattern and Optimal Funding Path from Current Funding Position

The first two examples illustrated funding strategies for new public pension systems. However, most public pension plans have been in operation for many years. Existing public pension plans may resemble mature systems with stable blocks of employees, such as the steady state achieved in the examples after 40 years, or the plans might not yet have achieved a steady state. Regardless of the growth pattern, each pension system currently has some level of liabilities and assets whether the funding ratio is optimal or not.

The current funding status of public pension systems varies widely from state to state. In this section, optimal funding paths will be illustrated on a by-state basis, starting with recent (1992) funding levels for each state and using individual state demographic trends.

The same assumptions are used here as in the previous examples, except that the pension system is assumed to have a 40-year history and is currently funded at some specified level. Thus, at time 0, the plan has accumulated liabilities and accumulated assets, $L(0)$ and $A(0)$, and the funding ratio ($x(0)$) is $A(0)/L(0)$. In this case, the accumulated liabilities at time 0 are much greater than those under the fully employed system discussed in the previous section because the liability for retirees also represents a part of the initial accumulated liability. The pension plan is to be funded in full within a specified period, in this case 80 years. As in the previous examples, the pension system will terminate in the final period by paying all the outstanding liabilities.

Based on the above assumptions, accumulated liabilities at the beginning of the ongoing plan are equal to 40th-year liabilities from the beginning of the gradual employment pattern, and accumulated assets are the current funding ratio times total liabilities. Since the system is already stabilized, the annual accrued benefit and payment streams increase at a rate of $(1+d')(1+g')$ over the years. That is,

$$D(t+1) = (1+d')(1+g')\,D(t), \text{ and } P(t+1) = (1+d')(1+g')\,P(t), \text{ for } t = 1, 2, 3, \ldots$$

The constant tax rate (τ) is determined such that the sum of the present value of the pension taxes plus the initial pension assets equal the sum of the present value of the accrued pension benefits and the initial accumulated liabilities. In this manner, the pension plan will have a funding ratio of one by the final period. Thus, the tax rate can be expressed as follows:

$$A(0) + \sum_{k=1}^{T} \frac{C(k)}{[1+r]^k} = \sum_{k=1}^{T} \frac{P(k)}{[1+q]^k} + L(0)$$

therefore

$$\tau = \frac{\displaystyle\sum_{k=1}^{T} \frac{P(k)}{[1+q]^k} + L(0) - A(0)}{\displaystyle\sum_{k=1}^{T} \frac{N(k)Y(k)}{[1+r]^k}}. \tag{15}$$

Once the tax rate and other parameters are determined, accumulated assets and liabilities are obtained based on equations (8) and (9); the optimal funding ratio is determined from these values. Table 2 represents the optimal funding path over the next 80 years using the U.S. national average data and a typical current funding level for the parameter values. The optimal funding path begins at 0.8 (the midpoint of the range of 1992 funding levels based on Dulebohn (1995)) and increases continuously, approaching 1.0 at the final period. Since the pension growth and tax base growth rates are very close ($(1+g')(1+d') = 1.0716$ vs. $(1+g)(1+d) = 1.0709$), the funding ratio increases at a fairly constant rate (0.002 to 0.004 per year) each year. However, the optimal path is not a straight line in all circumstances. Figure 3 illustrates the optimal funding paths under three different tax base growth rates, holding the pension growth rate constant. In this case, just a one percentage point change in the population growth rate from the U.S. average value makes a significant difference in optimal funding paths. When the tax base grows more rapidly than the pension benefit, the optimal funding path first declines from the current level of 0.8, before increasing back to 1.0 in the final year. Conversely, when the pension benefit grows more rapidly, the optimal funding path quickly increases from the current level to more than 1.0, and then declines to 1.0 by the final period. Figure 3 indicates that the optimal funding path is very sensitive to the relative growth rates of tax base and pension obligation. This relationship is examined more fully in the next section.

PENSION GROWTH RATE AND TAX BASE GROWTH RATE

The multi-period optimal funding model indicates that the relationship between the pension growth rate and the tax base growth rate is crucial in determining the optimal funding decision. When the tax base grows more rapidly than the pension obligation, even low funding in earlier periods is not a major problem since the underfunded portion can be financed by later generations with a larger tax base. Under the reverse situation, however, a public pension plan needs to be fully funded or over-funded in earlier periods to meet the rapidly growing pension cost. Otherwise, the cost of funding these benefits will be much higher for future generations, leading to the potential for encouraging relocation out of the state or producing the recessionary effects attributed to higher taxes.

Forecasting these relative growth rates is not easy. On a national basis, population and per capita income growth rates are relatively stable; however, they vary significantly from state to state. Pension growth is even more difficult to forecast. When a state's tax base is growing rapidly from an increasing population and rising per capita income, the number and the salary of public employees is likely to increase, but generally not as fast as the growing economy. In this case, the tax base grows more rapidly than pension costs. When a state's economy contracts, with a declining population and a slowing of per capita income growth, the number of public employees may not decrease in line with the population decline. Taxpayers may want to maintain the same level of public service, and public employees' job security may inhibit commensurate downsizing. In this case, pension costs would grow faster than the tax base.

Although the past is not always indicative of future trends, examining historical data is somewhat helpful for forecasting future patterns of the growth of the economy and the public sector. According to historical data, during the period from 1980 through 1992 the tax base and pension costs grew almost at a same rate, based on U.S. national

TABLE 2
Projected Optimal Funding Ratio Under Ongoing Pattern
Annual Growth Rates (1980–1992)

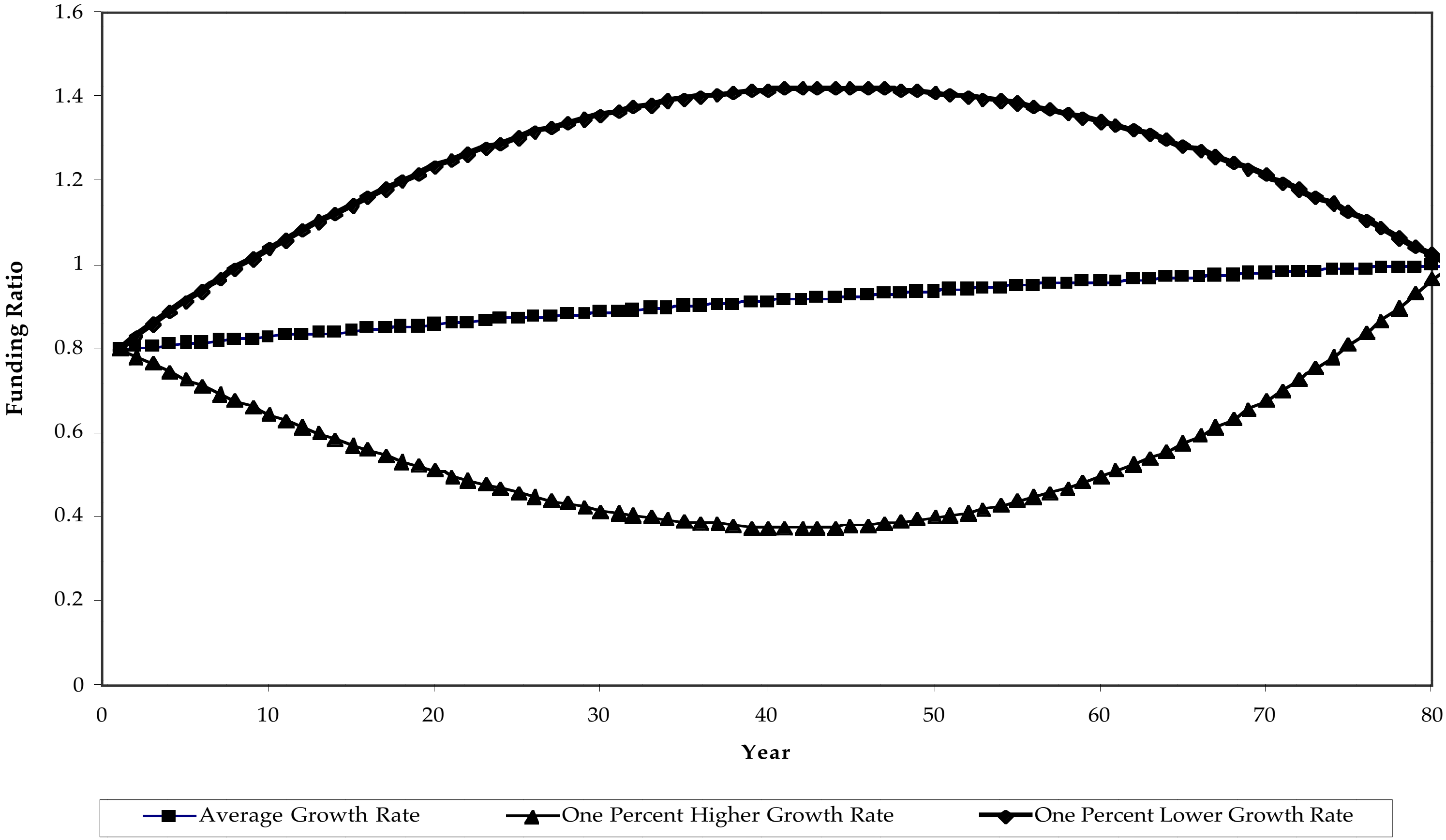
Population	1.00993		
Per Capita Income	1.06046	Salary	1.05536
Discount Rate	1.08	# of Employees	1.01539
Interest Rate	1.08	Current Funding	0.8

Year	*Payment*	*Accumulated Liabilities Workers*	*Accumulated Liabilities Retirees*	*Accumulated Liabilities Total*	*Pop.*	*Income*	*Tax*	*Accumulated Assets*	*Funding Ratios*
Initial Year (40)				851		80% of Liabilities		680	0.800
41	66	456	455	911	1.00	1.00	58	732	0.803
42	70	489	488	977	1.01	1.06	62	788	0.806
43	75	524	523	1047	1.02	1.12	67	847	0.809
44	81	562	560	1122	1.03	1.19	71	911	0.813
45	87	602	600	1202	1.04	1.26	77	980	0.816
46	93	645	643	1288	1.05	1.34	82	1055	0.819
47	99	691	689	1380	1.06	1.42	88	1134	0.822
48	107	740	738	1479	1.07	1.51	94	1220	0.825
49	114	793	791	1585	1.08	1.60	101	1312	0.828
50	122	850	848	1698	1.09	1.70	108	1412	0.831
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
77	792	5501	5487	10988	1.43	8.28	687	9969	0.907
78	848	5895	5880	11775	1.44	8.78	736	10714	0.910
79	909	6318	6301	12618	1.46	9.31	788	11513	0.912
80	974	6770	6752	13522	1.47	9.87	844	12372	0.915
81	1044	7255	7235	14490	1.48	10.47	904	13294	0.917
82	1119	7774	7753	15527	1.50	11.10	969	14285	0.920
83	1199	8331	8308	16639	1.51	11.77	1037	15349	0.922
84	1285	8927	8903	17831	1.53	12.48	1111	16492	0.925
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
116	11746	81618	81400	163018	2.10	81.67	9973	161891	0.993
117	12587	87462	87228	174691	2.12	86.61	10681	173791	0.995
118	13488	93725	93474	187199	2.14	91.85	11439	186561	0.997
119	14453	100436	100167	200603	2.16	97.40	12251	200263	0.998
120	230455	107627	107339	0	2.18	103.29	13121	0	1.000

Note: For the scaling purpose, population and income start from 1 instead of 100 as in the previous case.

data, although the respective components did not grow at the same rate. As stated previously, the U.S. population increased at a rate of 0.993 percent per year and per capita personal income increased 6.046 percent per year, while the number of state public employees increased at a rate of 1.539 percent per year and average salary at 5.536 percent per year. Even though these growth rates are different from each other, the tax base (population times per capita personal income) growth rate (1.0710) and pension (number of public employee times average salary) growth rate (1.0716) are almost same over the years from 1980 through 1992 on a national average.

FIGURE 3
Optimal Funding Paths for Different Tax Base Growth Rates



As far as individual states are concerned, however, these ratios differ widely as variations among states result in different economic and demographic situations. Table 3 represents the growth rates of population, per capita personal income, number of state public employees, and average salary by state. As shown in Table 3, the annual population growth rate of Nevada is 4.366 percent, while that of District of Columbia is –0.720 percent. The growth rate of per capita personal income is relatively stable compared to population growth rate, ranging only from Alaska's 4.057 percent to New Jersey's 6.951 percent. Similarly, the growth rate of the number of state public employees also varies by state. Even though the population growth rate and public employee growth rate are positively correlated ($r = 0.68$), the growth rates of some states diverge. For instance, North Dakota has negative population growth rate (–0.246 percent) while the number of public employees grew 1.421 percent per year during the same period. Average salary growth rates by state are also positively correlated with per capita income growth rate ($r = 0.48$) and are relatively stable, ranging from a minimum of 2.831 percent in Wyoming to a maximum of 8.652 percent in Connecticut. Table 4 lists the descriptive statistics of the growth rates of population, public employees, per capita personal income, and average monthly salary of the public employees by state.

TABLE 3
Tax Base and Pension Growth Rates by State (1980-1992)

	Population Growth Rate (%)	*Personal Income Growth Rate (%)*	*Public Employee Growth Rate (%)*	*Average Salary Growth Rate (%)*	*Tax Base Growth Rate (%)*	*Pension Growth Rate (%)*
Alabama	0.508%	6.620%	1.847%	4.786%	7.16%	6.72%
Alaska	3.220	4.057	2.781	3.873	7.41	6.76
Arizona	2.904	5.386	2.494	4.420	8.45	7.02
Arkansas	0.385	6.467	1.587	5.950	6.88	7.63
California	2.245	5.153	1.952	5.174	7.51	7.23
Colorado	1.524	5.708	0.675	5.491	7.32	6.20
Connecticut	0.447	6.915	0.160	8.652	7.39	8.83
Delaware	1.268	5.951	1.374	5.710	7.30	7.16
Washington, D.C.	–0.720	6.917	1.095	4.965	6.15	6.11
Florida	2.742	5.965	3.571	4.888	8.87	8.63
Georgia	1.807	6.874	2.273	4.720	8.81	7.10
Hawaii	1.516	6.210	2.604	5.119	7.82	7.86
Idaho	1.018	5.832	1.919	4.387	6.91	6.39
Illinois	0.135	5.956	0.592	5.121	6.10	5.74
Indiana	0.252	5.916	2.200	4.978	6.18	7.29
Iowa	–0.323	5.747	0.137	6.492	5.41	6.64
Kansas	0.517	5.824	1.914	4.386	6.37	6.38
Kentucky	0.209	6.177	0.792	5.913	6.40	6.75
Louisiana	0.143	5.199	0.178	5.563	5.35	5.75
Maine	0.787	6.832	1.300	5.609	7.67	6.98
Maryland	1.288	6.585	0.032	6.291	7.96	6.33
Massachusetts	0.364	6.877	0.203	6.705	7.27	6.92
Michigan	0.153	5.627	0.360	5.653	5.79	6.03

	Population Growth Rate (%)	*Personal Income Growth Rate (%)*	*Public Employee Growth Rate (%)*	*Average Salary Growth Rate (%)*	*Tax Base Growth Rate (%)*	*Pension Growth Rate (%)*
Minnesota	0.768	6.182	0.514	5.797	7.00	6.34
Mississippi	0.306	6.166	1.253	5.500	6.49	6.82
Missouri	0.453	6.162	1.194	4.447	6.64	5.69
Montana	0.363	5.304	0.197	4.637	5.69	4.84
Nebraska	0.163	6.424	0.173	5.487	6.60	5.67
Nevada	4.366	5.368	3.643	5.154	9.97	8.99
New Hampshire	1.606	6.941	0.752	5.787	8.66	6.58
New Jersey	0.501	6.951	2.217	6.754	7.49	9.12
New Mexico	1.630	5.482	1.289	4.948	7.20	6.30
New York	0.258	6.829	1.645	6.770	7.10	8.53
North Carolina	1.260	6.923	1.411	5.303	8.27	6.79
North Dakota	–0.246	6.916	1.421	4.457	6.65	5.94
Ohio	0.170	5.747	0.819	6.109	5.93	6.98
Oklahoma	0.483	4.844	1.448	4.611	5.35	6.13
Oregon	1.014	5.431	0.714	5.336	6.50	6.09
Pennsylvania	0.092	6.294	0.563	5.746	6.39	6.34
Rhode Island	0.463	6.451	–0.510	6.481	6.94	5.94
South Carolina	1.201	6.566	1.615	4.522	7.85	6.21
South Dakota	0.203	6.925	0.439	4.730	7.14	5.19
Tennessee	0.756	6.817	1.236	5.199	7.62	6.50
Texas	1.828	5.372	2.883	5.252	7.30	8.29
Utah	1.806	5.772	3.399	3.345	7.68	6.86
Vermont	0.929	6.787	0.783	5.989	7.78	6.82
Virginia	1.501	6.456	1.054	5.168	8.05	6.28
Washington	1.841	5.887	1.726	5.062	7.84	6.88
West Virginia	–0.624	5.753	–1.755	5.246	5.09	3.40
Wisconsin	0.495	5.715	0.630	6.102	6.24	6.77
Wyoming	–0.089	4.212	1.491	2.831	4.12	4.36
Total	0.993%	6.046%	1.539%	5.536%	7.10%	7.16%

Sources: Statistical Abstract of the United States and Public Employment

TABLE 4
The Basic Statistics of Growth Rates by State

	Population	*Public Employee*	*Per Capita Income*	*Average Salary*
Mean	0.993%	1.539%	6.046%	5.536%
Standard Deviation	0.983%	1.038%	0.695%	0.929%
Maximum	4.366%	3.643%	6.951%	8.652%
Median	0.508%	1.847%	6.620%	4.786%
Minimum	–0.720%	–1.755%	4.057%	2.831%
Correlation Coefficient	0.679		0.477	

Following the practice of most pension plans, it is assumed that pension benefits are determined based on the final salary and the number of years of public service rendered. Thus, pension liabilities increase as the number of covered public employees and their average salaries change each year. The growth of actual pension costs, however, may differ from the growth of the number of covered employees and their average salaries. For example, benefit improvements may increase pension costs at a greater rate than would be reflected by employee and salary growth.

In a continuing pension plan the benefits paid out during any year are not included as the pension cost. The real cost of a pension plan during any year is the liability incurred by covered employees for benefits they are expected to receive in the future as a result of their service and earnings during the current year. One popular measure of the pension cost is the Governmental Accounting Standards Board (GASB) Pension Benefit Obligation (PBO) which is computed using the projected unit credit actuarial method (GASB (1986)). The PBO is a standardized disclosure measure of the present value of pension benefits, adjusted for the effects of projected salary increases, estimated to be payable in the future as a result of employee service to date.

The PBO of state public employee pension plans for the years 1988 and 1992 are shown in Table 5 (these data were not available for prior years). The national average annual growth rate of the PBO over the period is 8.86 percent, which exceeds both the tax base growth rate of 6.16 percent for the same period and the pension growth as measured by the number of public employees and average salary for the same period of 8.08 percent. On a state basis, the differences are, in some cases, much larger. For example, the annual PBO growth in New Hampshire was 16.44 percent and in Mississippi 16.04 percent.

Pension cost increases can be the result of larger employment, increased salaries, or improvements in retirement benefits. Differences can be introduced by the assumptions used in calculating the PBO such as projected salary increases and benefit increases for active or retired employees.[7] Also, the total number of state public employees does not exactly match the number of public employees covered by the pension plan. Therefore, some differences exist between those two growth rates. In this case, the PBO growth rate is slightly greater than the growth rate of the number of public employees multiplied by average salary. However, as shown in earlier examples, even a small difference can have a major effect over an extended period.

These data demonstrate that public pension liabilities grew much faster than the tax base for the years from 1988 through 1992. This situation indicates that a higher pension funding level is necessary to cope with the growing pension obligations if the trend is assumed to continue.

PROJECTED OPTIMAL FUNDING RATIOS

Based on the model, the optimal funding levels of the public pension plans for each state are projected for 80 years. Table 6 represents the current (1992) funding ratios and the optimal funding ratios by state after 10 and 40 years when the pension fund-

[7] In the case of State Universities Retirement System of Illinois, assumptions used include projected salary increases of 4.5 percent per year compounded annually, attributable to inflation and an additional projected salary increase of 2.5 percent per year, attributable to seniority and merit. In addition, the benefits for retirees increase 3 percent annually.

Table 5
Pension Benefit Obligation Growth (1988–1992) by State

State	*PBO (1988)*	*PBO (1992)*	*Annual Growth Rate (%)*
Alabama	6749083	9972048	10.25%
Alaska	3176335	5414550	14.26
Arizona	6337655	9452836	10.51
Arkansas	2996810	4443310	10.35
California	88262573	115597800	6.98
Colorado	7863928	11335282	9.57
Connecticut	10476500	13420400	6.39
Delaware			
Washington, D.C.	6140212	7130900	3.81
Florida	23429000	37888543	12.77
Georgia	11732066	15990500	8.05
Hawaii	4361051	6092482	8.72
Idaho	1791500	2665800	10.45
Illinois	19453067	30131696	11.56
Indiana	2521018	3645325	9.66
Iowa	3827020	5597573	9.97
Kansas	3151116	4082000	6.68
Kentucky	5802928	8811597	11.01
Louisiana	11408264	14612633	6.38
Maine			
Maryland	14142576	18671653	7.19
Massachusetts			
Michigan	17143457	25410417	10.34
Minnesota	10832052	16098535	10.41
Mississippi	4109718	7452077	16.04
Missouri	6537957	10368988	12.22
Montana	1112900	1522600	8.15
Nebraska			
Nevada	4467605	5725504	13.21
New Hampshire	870700	1600500	16.44
New Jersey	21117263	33166194	11.95
New Mexico			
New York	61525900	82864700	7.73
North Carolina	11722010	15597361	7.40
North Dakota	645300	957600	10.37
Ohio	41583997	57522960	8.45
Oklahoma	4513189	6860048	11.04
Oregon	10025200	12660700	6.01
Pennsylvania	26343500	31415415	4.50
Rhode Island			
South Carolina	7991800	11600700	9.76
South Dakota	1011600	1519100	10.70
Tennessee	6376100	9331700	9.99
Texas	26644314	39347775	10.24
Utah	3572726	5359032	10.67
Vermont	319600	498096	11.73
Virginia	10569100	15771300	10.52
Washington	15475719	22016200	9.21
West Virginia			
Wisconsin	14894200	22818000	11.25
Wyoming	1149791	1731357	10.78
Total	544178400	764173787	8.86%

Source: Annual Reports of Public Employee Pension Systems by State ing horizon is assumed to be 80 years.[8] State employment growth (SEG) rates (salary times number of public employees) and PBO growth rates are each used as the pension growth rate. Different funding ratios and patterns of funding over time were obtained depending on the parameter values for the states. The funding patterns result from the current funding level and the relative magnitude of tax base and pension growth rates during the period.

The results shown in Table 6 vary drastically, with optimal funding levels after 10 years ranging from 29.9 percent (in Maine, based on the state employment growth rates) to 4671.7 percent funding (in Mississippi, based on the PBO growth rates). For 40-year levels, New Hampshire indicates a value of 5 percent (based on state employment growth), Virginia indicates a value of –4.4 percent (also based on state employment growth), and the District of Columbia indicates a value of –91.5 percent (based on the PBO growth rates). These startling results are not due to a problem with the model, but demonstrate, in convincing fashion, the sensitivity of the optimal funding levels to the relationship between pension growth rates and the tax base growth rate.

For Maine, the tax base growth rate was 7.67 percent (Table 3), whereas the pension growth rate based on the state employment growth was 6.98 percent (also Table 3); since the tax rate exceeds the pension growth rate, underfunding is the optimal strategy. In Mississippi, on the other hand, the tax base growth rate was 6.49 percent (Table 3) but the pension growth rate was 16.04 percent (Table 5); since the pension growth rate is so much higher than the tax growth, the optimal funding strategy requires significant prefunding. If the funding strategy proposed here were universally adopted, then the resulting indications of the need for such significant advance funding would likely serve as a restraint of the rapid growth in pension benefits, minimizing future financial problems for public pension funds. Also, use of this standard would also serve to deter pressure for additional benefit increases on plans that are appropriately funded in excess of 100 percent.

For the 40-year levels, the effects of a two to three percentage point difference between the tax growth rate and the pension growth rate are demonstrated. In New Hampshire, the low optimal funding level occurred since the tax base growth rate of 8.66 percent (Table 3) exceeded the pension growth rate of 6.58 percent (also Table 3) by so much. In Virginia similar values, 8.05 percent for the tax growth and 6.28 percent for pension growth (Table 3), led to an impractical negative funding level for the optimal value. In the District of Columbia, the tax base growth rate was 6.15 percent (Table 3) and the pension growth rate only 3.81 percent (Table 5), leading to another negative value.

Although it is unlikely that the growth rates for taxes and pensions will diverge by much over an extended period of time, these values are difficult to predict. The data illustrated in Tables 3 and 5 demonstrate actual changes over the recent past, based on two different sources of data. The values in Table 6 demonstrate the sensitivity of optimal funding levels to the relevant growth rates, illustrating that care should be taken to develop realistic values for these parameters.

[8] The funding ratios were calculated using Dulebohn (1995) data (referenced here as SURS). For the states not included, PENDAT data were used (Zorn (1994)). A comparison of the ratios from the two sources indicates that they are very similar. Massachusetts and West Virginia are excluded because of missing data.

TABLE 6
Current & Optimal Funding Ratios After 10 and 40 Years by State
(State Employment Growth (SEG) and Pension Benefit Obligation Growth (PBO) are used)

	Current Funding Ratio		*Optimal Funding Ratio*		*After 10 and 40 Years*	
	(1992)		*10*	*40*	*10*	*40*
State	*Pendat*	*SURS*	*SEG*		*PBO*	
Alabama	1.017	1.047	0.970	0.832	2.415	2.903
Alaska	0.897	0.894	0.793	0.638	14.095	8.844
Arizona	NA	1.180	1.099	0.955	1.981	2.303
Arkansas	0.996	0.988	1.157	1.364	2.645	3.15
California	0.969	0.940	0.890	0.827	0.853	0.721
Colorado	1.075	1.075	0.905	0.557	1.853	2.288
Connecticut	0.654	0.582	1.116	1.685	0.439	0.231
Delaware	1.095	NA	1.061	0.993		
Washington, D.C.	0.301	0.334	0.369	0.537	0.019	–0.915
Florida	0.759	0.759	0.739	0.769	4.078	4.133
Georgia	0.973	1.082	0.806	0.271	0.926	0.685
Hawaii	NA	0.753	0.791	0.892	1.034	1.369
Idaho	0.697	0.788	0.715	0.619	2.597	3.195
Illinois	0.631	0.630	0.589	0.568	5.124	5.034
Indiana	NA	1.178	1.379	1.581	2.528	2.999
Iowa	1.007	1.112	1.329	1.586	3.249	3.662
Kansas	1.010	1.076	1.073	1.054	1.121	1.175
Kentucky	0.867	0.860	0.939	1.081	3.814	4.135
Louisiana	NA	0.561	0.656	0.885	0.787	1.182
Maine	0.389	NA	0.299	0.253		
Maryland	NA	0.770	0.671	0.154	0.652	0.499
Michigan	0.697	0.852	0.901	1.008	3.302	3.786
Minnesota	0.933	0.887	0.785	0.615	2.567	3.123
Mississippi	0.794	0.796	0.878	1.035	46.717	17.225
Missouri	0.987	1.019	0.889	0.617	6.345	5.602
Montana	0.845	0.883	0.780	0.561	1.558	2.115
Nebraska	0.938	NA	0.807	0.543		
Nevada	0.799	0.799	0.612	0.416	3.385	3.516
New Hampshire	NA	1.033	0.722	0.050	29.965	13.220
New Jersey	0.824	0.929	1.469	1.902	4.399	4.438
New Mexico	0.780	NA	0.644	0.420		
New York	0.984	1.082	1.467	1.792	1.202	1.329
North Carolina	1.217	1.198	0.959	0.467	1.033	0.730
North Dakota	1.072	1.105	1.000	0.783	2.896	3.338

TABLE 6 , CONTINUED

	Current Funding Ratio(%)		*Optimal Funding Ratio (%) after 10 and 40 Years*			
	(1992)		*10*	*40*	*10*	*40*
State	*Pendat*	*SURS*	*SEG*		*PBO*	
Ohio	0.967	0.869	1.099	1.416	1.615	2.190
Oklahoma	0.565	0.462	0.647	1.016	4.673	4.815
Oregon	1.100	1.102	1.034	0.896	1.029	0.880
Pennsylvania	0.995	0.973	0.966	0.959	0.781	0.261
Rhode Island	1.437	NA	1.287	0.921		
South Carolina	0.743	0.769	0.529	0.048	1.509	2.038
South Dakota	1.174	1.174	0.953	0.371	2.967	3.342
Tennessee	1.163	1.163	0.983	0.615	2.039	2.431
Texas	1.073	1.061	1.298	1.527	2.332	2.808
Utah	0.978	0.769	0.646	0.469	2.284	2.873
Vermont	NA	0.812	0.654	0.402	3.590	3.901
Virginia	0.782	0.763	0.497	–0.044	1.916	2.489
Washington	0.792	0.774	0.621	0.385	1.238	1.671
Wisconsin	1.151	1.067	1.159	1.278	4.546	4.568
Wyoming	1.118	1.112	1.131	1.158	5.634	5.174
Total	0.917	0.922	0.936	0.973	1.453	1.892

Note: SURS ratios are used as current funding. For the states SURS ratios are not available, Pendat is used.

The following states use book value as an asset value: Arkansas, Connecticut, Minnesota, and Utah. Massachusetts and West Virginia are excluded because of missing data. In addition, Delaware, Maine, Nebraska, New Mexico, and Rhode Island are excluded when PBO growth rate is used as a pension growth rate.

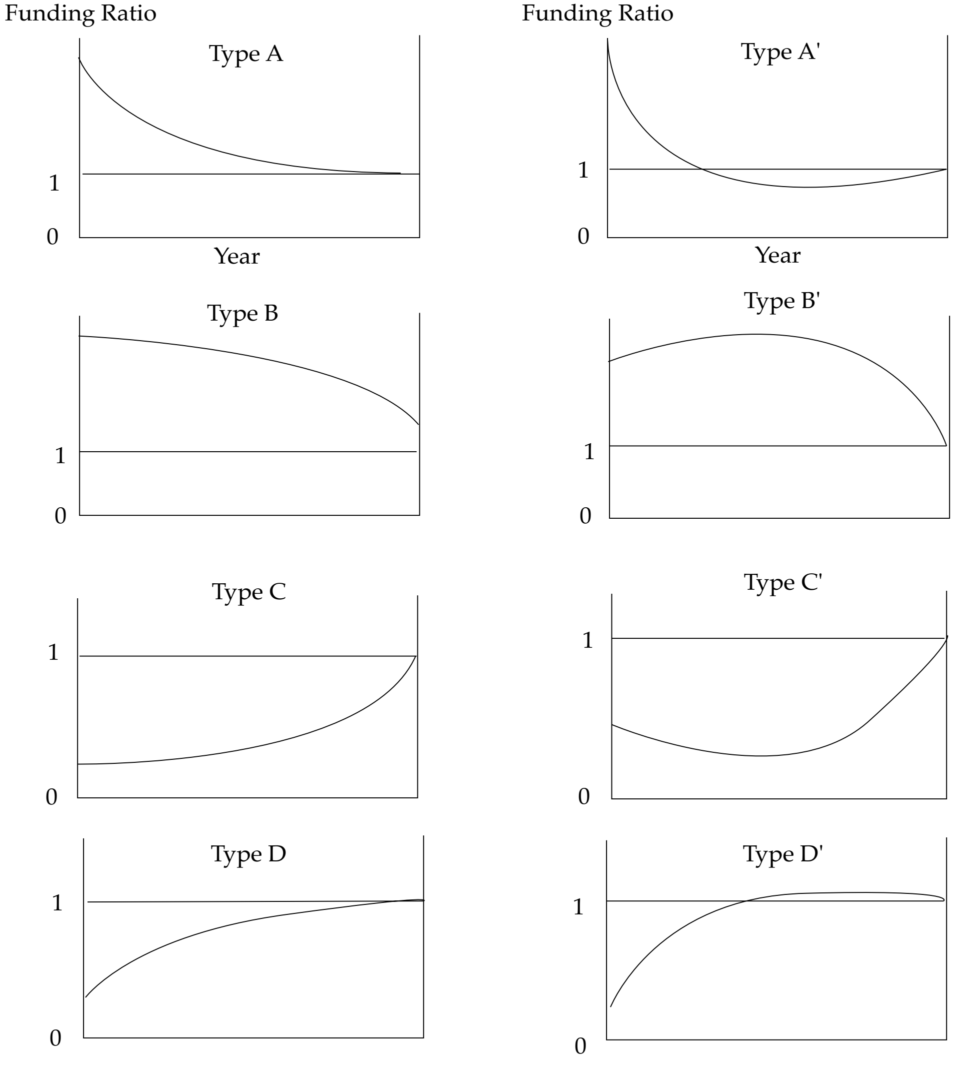
ILLUSTRATION OF OPTIMAL FUNDING PATHS

The current funding can be either above or below 1.0. Based on this model, the funding level will move from the current level to 1.0 over time. The shape of this curve will either be convex or concave, depending on the relationship between the growth rates for tax base and pensions.

For the case where the current funding level is above 1.0, if the shape is concave or convex enough, then the curve will move: for a concave curve, above the current level before converging on 1.0, or, for a convex curve, below 1.0 before converging to 1.0. Thus, for the situation where the current level is above 1.0, there are four possible paths of convergence.

Similarly, there are four possible paths for convergence when the current funding level begins below 1.0. For a convex curve, the curve can be slightly convex, moving consistently upward from the original level to 1.0, or it can be convex enough so that the funding level falls below the original level before increasing. For a concave curve,

FIGURE 4
Eight Different Types of Projected Optimal Funding Patterns



the curve can go consistently upward to 1.0, or it can be bowed so much that it goes above 1.0 before converging to 1.0. Thus, there are eight possible optimal funding paths in total. These are depicted in Figure 4.

Type A represents the case where the funding ratio begins above 1.0 and the tax base growth rate is higher than the pension growth rate. In this case, the optimal funding path follows a convex curve directly to 1.0. Type A' is similar to Type A, except that the path is so convex that it falls below 1.0 before converging.

Type B represents the case where the funding ratio begins above 1.0, but the pension growth rate exceeds the tax base growth rate. In this case, the optimal funding path follows a concave curve directly to 1.0. Type B′ is similar to Type B, except that the path is so concave that it rises above the original funding level before converging on 1.0.

Type C represents the case where the funding ratio begins below 1.0, but the tax base growth rate exceeds the pension growth rate. In this case, the optimal funding path follows a convex curve gradually rising to 1.0. Type C′ is similar to Type C, except that the path is so convex that it falls below the original funding level before converging to 1.0.

Type D represents the case where the funding ratio begins below 1.0 and the pension growth rate exceeds the tax base growth rate. In this case, the optimal funding path follows a concave curve directly to 1.0. Type D′ is similar to Type D, except that the path is so concave that it rises above 1.0 before converging.

Since the pension growth rate is such an important factor in determining the optimal funding patterns, both the state employment growth rate and PBO growth rate are used as a pension growth rate to classify the states by type, as shown in Table 7. The states in the best funding position are those in Type A or A′, where the current funding is in excess of 1.0 and the tax base growth rate exceeds the pension growth rate. When the state employment growth rate is used as a pension growth rate, thirteen states belong to Type A′, but no states are Type A. Based on the PBO growth rates, only 5 states belong to Type A′, and none to Type A. On the other extreme, Types D and D′ represent the worst funded states since the pension plans are poorly funded and the pension costs are growing much faster than the economy. Type D′ is even worse than Type D since the gap between the pension growth and tax base growth rates is larger. Based on the state employment growth, two states belong to Type D and eight states belong to Type D′. Based on the PBO growth rates, no states belong to Type D, but 20 states belong to Type D′. Type B and B′ represent the states where public pension plans are currently well funded but the pension cost is growing faster than the economy. Type C and C′ represent states where public pension plans are not fully funded but the economy is growing faster than pension costs.

Determining which type is better in terms of pension funding depends on the relative magnitude of the growth rates and how underfunded the plans currently are. In the case of Type B′, for example, even though the current funding status of the public pension plans is good, the state needs to raise the funding ratio even higher than the current level since the pension cost is growing much faster than the tax base. Six states belong to this type based on the state employment growth rates, and 16 states based on the PBO growth rates. New York is a typical example. On the contrary, even though the plans are not well funded based on the conventional approach, the taxpayers in the states of Type C′ do not need to worry as much about the current funding status since the economy is growing fast enough to meet the growing pension cost. Based on state employment growth, eighteen states belong to this category, including Illinois and California. Based on the PBO growth rates, only eight states fall into this category. Washington, D.C., belongs to Type C based on the state employment growth rates. Even though the tax base is growing a little faster than pension costs, it is necessary to improve the funding status continuously since the difference in the growth rates is so small (1.0615 vs. 1.0611) and the current funding level is so low (0.334). Examining which state belongs to which type can help determine the optimal funding pattern for future years from the current funding position of the states.

Table 7
Eight Different Types of Optimal Funding Patterns

Type	Characteristics	States in this Type Based on SEG	States in this Type Based on PBO
A	Current funding over 1.0; Convex curve converges directly to 1.0	None	None
A′	Current funding over 1.0; Convex curve increases, then converges to 1.0	Alabama, Arizona, Colorado, Delaware, Georgia, Missouri, New Hampshire, North Carolina, North Dakota, Oregon, Rhode Island, South Dakota, Tennessee	Delaware, Georgia, North Carolina, Oregon, Rhode Island
B	Current funding over 1.0; Concave curve converges directly to 1.0	Kansas	None
B′	Current funding over 1.0; Concave curve falls below 1.0 before converging to 1.0	Indiana, Iowa, New York, Texas, Wisconsin, Wyoming	Alabama, Arizona, Colorado, Hawaii, Indiana, Iowa, Kansas, Missouri, New Hampshire, New York, North Dakota, South Dakota, Tennessee, Texas, Wisconsin, Wyoming
C	Current funding below 1.0; Convex curve converges directly to 1.0	Washington, D.C.	None
C′	Current funding below 1.0; Convex curve decreases, then converges to 1.0	Alaska, California, Florida, Idaho, Illinois, Maine, Maryland, Minnesota, Montana, Nebraska, Nevada, New Mexico, Pennsylvania, South Carolina, Utah, Vermont, Virginia, Washington	California, Connecticut, Washington, D.C., Maine, Maryland, Nebraska, New Mexico, Pennsylvania
D	Current funding below 1.0; Concave curve converges directly to 1.0	Hawaii, Louisiana	None
D′	Current funding below 1.0; Concave curve rises above 1.0 before converging to 1.0	Arkansas, Connecticut, Kentucky, Michigan, Mississippi, New Jersey, Ohio, Oklahoma	Alaska, Arkansas, Florida, Idaho, Illinois, Kentucky, Louisiana, Michigan, Minnesota, Mississippi, Montana, Nevada, New Jersey, Ohio, Oklahoma, South Carolina, Utah, Vermont, Virginia, Washington

The actual Pension Benefit Obligation growth rates of state public pension plans are different from the growth rates of the number of covered employees and their average salaries. Illustrating the sensitivity of the results to the pension growth rate, the optimal funding patterns of 25 of the 49 states change to different types when the PBO growth rate is used instead of the state employment growth rate. Most (23 of 25) of the shifted states belong to Type B′ and Type D′ since the PBO growth rate is greater than state employment growth rate. For the case of Illinois, for instance, the optimal funding pattern shifts from Type C′ to Type D′ since the PBO growth rate (11.56 percent) is much greater than the pension cost growth measured by the state employment growth (5.74 percent). If actual pension costs grow at the same rate in the future, Illinois, for example, needs to fund at a higher level in earlier years to meet the rapidly growing pension costs.

Conclusion

The optimal funding level of public pension plans depends on such factors as the state's current and future tax base, interest rates, and the utility of wealth over time. Current funding strategies tend to ignore these relationships and instead focus on the impact of pension funding on the current state budget. This strategy has led to a wide variety of funding levels, in many cases levels that are far from optimal. Focusing on the economic and demographic variables of the state, this study develops an optimal pension funding model that incorporates the relevant factors under various circumstances. The results indicate that the relationship between the pension growth rate and the tax base growth rate plays a crucial role in determining the optimal funding decision. Only if the growth in pension costs over time can be constrained below the growth in the tax base, are funding levels less than one ever optimal. However, if pension costs grow faster than the tax base then the optimal funding strategy requires overfunding of public pension plans.

Appendix A

Utility Maximization in Terms of Representative Taxpayers for T Years

The government wants to maximize the representative taxpayer's T period consumption after paying the pension tax. Thus, the objective function and constraint are as follows:

$$Max \quad \log[W_1(1-\tau_1)] + \log[W_2(1-\tau_2)] + \log[W_3(1-\tau_3)] + \cdots + \ln[W_T(1-\tau_T)]$$

subject to:

$$W_1\tau_1 + W_2\tau_2 + W_3\tau_3 + \ldots + W_T\tau_T = P$$

where

W_t = personal income period t

τ_τ = pension tax rate in period t

P = total pension payments for period T.

The basic economic property of this instantaneous utility function is that the elasticity of substitution between consumption at any point in time is constant and equal to one and utility is additively separable over time. This utility function is frequently used in intertemporal optimization model. (*Lectures on Macroeconomics* by Blanchard, Oliver J. and Stanley Fischer (1992))

If personal income grows at a constant rate g each year, the Lagrange multiplier is as follows:

$$\begin{aligned} L = {} & \ln[W(1-\tau_1)] + \ln[W(1+g)(1-\tau_2)] + \cdots + \ln[W(1+g)^{T-1}(1-\tau_T)] \\ & +\lambda\{[W\tau_1 + W(1+g)\tau_2 + W(1+g)^2\tau_3 + \cdots + W(1+g)^{T-1}\tau_T] - P\}. \end{aligned}$$

First-Order Conditions

$$\frac{\partial L}{\partial \tau_1} = \frac{-w}{w(1-\tau_1)} + w\lambda = 0 \Rightarrow \frac{1}{1-\tau_1} = \lambda$$

$$\frac{\partial L}{\partial \tau_2} = \frac{-w(1+g)}{w(1+g)(1-\tau_2)} + w(1+g)\lambda = 0 \Rightarrow \frac{1}{1-\tau_2} = \lambda$$

$$\frac{\partial L}{\partial \tau_T} = \frac{-w(1+g)^{T-1}}{w(1+g)^{T-1}(1-\tau_T)} + w(1+g)^{T-1}\lambda = 0 \Rightarrow \frac{1}{1-\tau_T} = \lambda$$

$$\frac{\partial L}{\partial \lambda} = W\tau_1 + W(1+g)\tau_2 + W(1+g)^2\tau_3 + \ldots + W(1+g)^{T-1}\tau_T = P.$$

Thus,

$$\frac{1}{1-\tau_1} = \frac{1}{1-\tau_2} = \cdots = \frac{1}{1-\tau_T} = \lambda \Rightarrow \tau_1 = \tau_2 = \cdots = \tau_T = 1 - \frac{1}{\lambda}$$

meaning utility is optimized when the tax rate is constant.

And, $W\tau[1+(1+g)+(1+g)^2+\ldots+(1+g)^{T-1}]=P$

$$\therefore W\tau\left[\frac{(1+g)^T-1}{g}\right]=P,\ \text{if } g\neq 0 \text{ and } W\tau T=P,\ \text{if } g=0.$$

The tax rate that optimizes utility is:

$$\tau=\frac{P}{WT},\ \text{if } g=0,\ \text{and } \tau=\frac{Pg}{W[(1+g)^T-1]},\ \text{otherwise.}$$

APPENDIX B

Accrued Liabilities Under Fully Employed System

1. After working one year, each block of employees earns 1/20th of the final salary, which will be paid for 20 years after retirement.
2. The number of public employees increases at a rate of d' percent each year.
3. The salary of public employees increases at a rate of g' percent each year.
4. The accrued benefit is the discounted present value of the stream of payments.

Therefore, one year after the pension plan begins, accrued benefits earned for that year by each block of employees are as follows:

For employees who already worked for 19 years:

$$\frac{S}{20}\,\frac{1}{1+\mathrm{q}}\left[1+\frac{1}{1+q}+\left(\frac{1}{1+q}\right)^2+\ldots+\left(\frac{1}{1+q}\right)^{19}\right].$$

For employees who already worked for 18 years:

$$\frac{S}{20}\,\frac{1}{1+\mathrm{q}}\left[\frac{1}{1+q}+\left(\frac{1}{1+q}\right)^2+\ldots+\left(\frac{1}{1+q}\right)^{20}\right].$$

For employees who already worked for 17 years:

$$\frac{S}{20}\,\frac{1}{1+\mathrm{q}}\left[\left(\frac{1}{1+q}\right)^2+\left(\frac{1}{1+q}\right)^3+\ldots+\left(\frac{1}{1+q}\right)^{21}\right].$$

For employees who have just started to work:

$$\frac{S}{20}\,\frac{1}{1+\mathrm{q}}\left[\left(\frac{1}{1+q}\right)^{19}+\left(\frac{1}{1+q}\right)^{20}+\ldots+\left(\frac{1}{1+q}\right)^{38}\right].$$

Accrued liabilities at the end of the first year, D(1), are the sum of the above 20 items. That is,

$$D(1) = \frac{S}{20}\frac{1}{1+q}\left\{\left[1+\frac{1}{1+q}+\left(\frac{1}{1+q}\right)^2+\cdots+\left(\frac{1}{1+q}\right)^{19}\right]+\cdots+\left[\left(\frac{1}{1+q}\right)^{19}+\left(\frac{1}{1+q}\right)^{20}+\cdots+\left(\frac{1}{1+q}\right)^{38}\right]\right\}$$

$$= \frac{S}{20}\frac{1}{1+q}\sum_{i=0}^{19}\sum_{j=0}^{19}\left(\frac{1}{1+q}\right)^{i+j} = \frac{S}{20}(1+q)\left[\frac{1-(1+q)^{-20}}{q}\right]^2.$$

APPENDIX C

Retroactive Liabilities on the Inception of the Plan Under Ongoing Employment Pattern

1. At the commencement date of the pension plan, every block of employees has earned pension benefits which will be paid after retirement proportionate to their working years.
2. The accrued benefit is the discounted present value of the stream of payments.
3. Therefore, retroactive liabilities for each block of employees are as follows:

For employees who work for 19 years:

$$\frac{19}{20}S\frac{1}{1+q}\left[\frac{1}{1+q}+\left(\frac{1}{1+q}\right)^2+\cdots+\left(\frac{1}{1+q}\right)^{20}\right].$$

For employees who work for 18 years:

$$\frac{18}{20}S\,\frac{1+d'}{1+q}\left[\left(\frac{1}{1+q}\right)^2+\left(\frac{1}{1+q}\right)^3+\cdots+\left(\frac{1}{1+q}\right)^{21}\right].$$

For employees who work for 1 year:

$$\frac{1}{20}S\,\frac{(1+d')^{18}}{1+q}\left[\left(\frac{1}{1+q}\right)^{19}+\left(\frac{1}{1+q}\right)^{20}+\cdots+\left(\frac{1}{1+q}\right)^{38}\right].$$

Retroactive liabilities at time 0, L(0), is the sum of the above 19 items. That is,

$$L(0) = \frac{S}{20}\left\{19\left[\left(\frac{1}{1+q}\right)^2 + \cdots + \left(\frac{1}{1+q}\right)^{21}\right] + 18(1+d')\left[\left(\frac{1}{1+q}\right)^3 + \left(\frac{1}{1+q}\right)^4 + \cdots + \left(\frac{1}{1+q}\right)^{22}\right] + \cdots\right.$$

$$\left. + (1+d')^{18}\left[\left(\frac{1}{1+q}\right)^{20} + \cdots + \left(\frac{1}{1+q}\right)^{39}\right]\right\} = \frac{S}{20}\left[\frac{1-(1+q)^{-20}}{q(1+q)}\right]\left[\frac{k(k^{19}-1)-19(k-1)}{(k-1)^2}\right],$$

where $k = \dfrac{1+d'}{1+q}$.

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